



Naval Research Laboratory

Washington, DC 20375-5320

NRL/MR/6707--97-7915

Theoretical Aspects of the Enhanced Glow Discharge

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May 16, 1997

19970527 113

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED	
	May 16, 1997	Interim Report	
4. TITLE AND SUBTITLE			5. FUNDING NUMBERS
Theoretical Aspects of the Enhanced Glow Discharge			
6. AUTHOR(S)			8. PERFORMING ORGANIZATION REPORT NUMBER
Wallace M. Manheimer and Richard Fernsler			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)			10. SPONSORING/MONITORING AGENCY REPORT NUMBER
Naval Research Laboratory Washington, DC 20375-5320			
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			12b. DISTRIBUTION CODE
Office of Naval Research Arlington, VA 22217-5660			
11. SUPPLEMENTARY NOTES			
12a. DISTRIBUTION/AVAILABILITY STATEMENT		Approved for public release; distribution unlimited.	
13. ABSTRACT (Maximum 200 words)			
The agile mirror plasma has the capability of redirecting microwave beams electronically. It is now well established that the plasma is a beam generated plasma, where the beam is generated by a hollow cathode discharge. A new hollow cathode mode of operation, called the enhanced glow mode has recently been established. The main difficulty in understanding the enhanced glow is the behavior of the cathode plasma. This work is an initial attempt at a theory of the cathode region of the enhanced glow plasma.			
14. SUBJECT TERMS			15. NUMBER OF PAGES
Agile mirror Hollow cathode Cathode fall			
			28
16. PRICE CODE			
17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

CONTENTS

I. INTRODUCTION	1
II. REVIEW OF THE CATHODE SHEATH FOR A PLANAR CATHODE	3
A. Ionization Only by Thermals	6
B. Ionization Only by the Beam	7
III. THE CATHODE REGION OF THE ENHANCED GLOW	9
A. The Uniform Plasma Assumption	9
B. The Langmuir-Child Ion Diode Including Beam Ionization	11
C. Approximate Enhanced Glow Relations	13
IV. THE POSSIBILITY OF NUMERICAL SIMULATIONS OF THE ENHANCED GLOW CATHODE	16
REFERENCES	21

THEORETICAL ASPECTS OF THE ENHANCED GLOW DISCHARGE

I. Introduction

The agile mirror has been under investigation at NRL for several years now¹⁻⁶. This is a planar sheet plasma which has the capability of reflecting microwave beams with the same quality (as regards for instance side lobes and emission temperature) as a flat metal plate. The fact that the position of the plasma can be repositioned electronically either by varying the direction of the confining magnetic field, or by designating the emission position on the cathode, means that a radar or other microwave system could be electronically steered using a very small, simple, lightweight and inexpensive system. As such, the agile mirror is of considerable potential interest to the Navy.

A great deal about the agile mirror plasma is now known. It is a beam generated plasma, where the beam is produced in a hollow cathode, propagates along the magnetic field and ionizes the background gas. This beam has been measured in Ref.(5), and there it was estimated that about 5% of the discharge current typically is beam current. The ionization by the beam is balanced by recombination in the relatively cool plasma. In almost all cases, the ions are diatomic, so dissociative recombination is important. (Even for monatomic gases, it may be that dimers such as A_2^+ form from metastables. In practice the behavior of argon plasmas is not very different from the behavior of air.) Diffusion is another important loss mechanism in the agile mirror plasma. Reference (5) points out the presence of a new plasma regime, the enhanced glow mode. It has much higher impedance than the standard hollow cathode mode, but lower impedance than the than the abnormal glow. It is capable of producing a high energy electron beam, and also of producing a plasma with high electron density. The electron densities in the enhanced glow mode are at least an order of magnitude higher than those in normal glow plasmas. As such, it is well suited to the needs of the agile mirror.

The main enigma concerning the enhanced glow is the operation of the hollow cathode region and the generation of the electron beam. This work is an initial analytic description of the physics of the enhanced glow plasma, and particularly a description of the behavior of the cathode. It attempts, and achieves some success, in describing the behavior of the enhanced glow cathode. Section 2 reviews the behavior of the conventional cathode fall. It turns out that this provides considerable insight into the cathode region of the enhanced glow cathode. Here the voltage, current and sheath width are determined by the collisional or collisionless ion diode law, whichever is appropriate. The sheath thickness is then related to the voltage and current by the additional relation that the ionization is just sufficient to convert the ion current at the cathode to electron current at the entrance to the sheath region. Section 3 works out a simple theory for the enhanced glow or hollow cathode mode assuming, that like the planer discharge, the high voltage, high current density sheaths are maintained a balance of ionization and the Langmuir Child space charge limited diode relations. However the ionization is now caused by electrons reflexing in the cathode hollow, back and forth between the sheaths. Furthermore, the ion diode law is modified by the ionization generated by the electron beam.

This theory give qualitative and even semi-quantitative insight into this plasma regime. Particularly it gives the current as a function of voltage, gas pressure, and cathode geometry. It also gives some qualitative insight into the dependence on the magnetic field. Our hope is that this qualitative analysis will open the way to more accurate numerical simulations (or better analytic theories). There has been a great deal of progress in numerical simulation of hollow cathode discharges⁷⁻¹². These simulation schemes also can describe normal flat plate cathodes. Section 4 discusses the potential application of such simulations to enhanced glow discharges. We find that they are generally applicable, but for best results, there are several modifications that could be made to the codes.

II Review of the Cathode Sheath for a Planar Cathode

A plasma in a dc discharge is characterized by a region of low electric field, where the plasma is quasi-neutral. Between this region and cathode is a sheath. This cathode sheath is much larger than the Debye length, and yet the plasma is not quasi-neutral in this region. In many experiments the electric fields are measured in this region and are found to vary approximately linearly in space between the cathode and quasi-neutral plasma, with the maximum field on the cathode¹³. The generally accepted explanation is that electrons are nearly excluded from this region, because the equilibrium electron density scales as $\exp(e\phi/T_e)$; for electron temperatures of an electron volt or two, and sheath potentials decreasing from zero to negative a few hundred volts in a centimeter or so, the electrons are excluded by the strong electrostatic fields.

The cathode sheath has well known scaling relations.^{13,14} The width of the sheath is proportional to the reciprocal of the background pressure, the current density is proportional to the pressure squared, and electric field is proportional to pressure. If one couples the Boltzmann equation for the charged particle distribution function to Maxwell's equations for the fields, and allows collisions only with the neutral background, it is not difficult to derive the preceding scaling relationships.

Another remarkable experimental fact regarding the cathode fall is that there is a minimum voltage below which no discharge is produced. As one attempts to lower this voltage, say by lowering the current, the voltage remains constant (if it is at the minimum value), the current density remains constant, but the emitting area on the cathode decreases. Finally, the ion current density going into the sheath is given by the Bohm current, $n_e c_s$, where n is the ion density and c_s is the ion acoustic velocity in the plasma. This Bohm condition comes from very fundamental conditions involving the joining of a region where quasi-neutrality is valid to one where it is not, i.e. a sheath region. Because we will contrast the Bohm condition with other possible conditions later on, we will now give a brief derivation of it.

Consider a plasma described by the electron and ion continuity equations, electron and ion momentum equations for isothermal plasmas with specified temperature, and Poisson's equation. Consider the ambient state to be a uniform plasma with electron and ion density n_o , flow velocity v_o , zero potential, and Maxwellian electrons. In the perturbed plasma, with nonzero electrostatic potential, the electron density as a function of potential ϕ , is $n_e \exp(e\phi/T_e)$. Next consider the ions. The ion density and momentum equations reduce in steady state to

$$d/dx (n_i v_i) = 0 \quad \text{or} \quad n_i v_i = \text{constant} \quad (1)$$

$$d/dx(n_i M v_i^2) = -n_i e d\phi/dx \quad (2)$$

where for convenience, we have considered cold ions. Linearizing the ion and electron equations, solving for the perturbed densities as function of ϕ and inserting these in Poisson's equation, we find Poisson's equation becomes

$$d^2\phi/dx^2 = k_D^2 [1 - T_e/Mv_i^2] \phi \quad (3)$$

where $k_D^2 = 4\pi e^2 n_e / T_e$. If $v_i < [T_e/M]^{1/2}$, the solution to Poisson's equation oscillates in space about the ambient value, and the plasma remains quasi-neutral. If $v_i > [T_e/M]^{1/2}$, the solution to Poisson's equation grows exponentially in space, meaning that the plasma is no longer quasi-neutral, but forms a charge rich sheath. Thus if $v_i < [T_e/M]^{1/2}$, the plasma is a quasi-neutral bulk plasma. The plasma flows into the wall, but does so slowly enough that quasi-neutrality can be maintained. The position where $v_i = [T_e/M]^{1/2}$, marks the position where the flow is so fast that quasi-neutrality cannot be maintained, the onset of the sheath. This is the Bohm condition. Finally, let us remark that in either case, the scale length for the oscillation or exponential growth is roughly the Debye length, a very small distance for almost all plasmas. Thus even if the plasma is collisional, as long as the collision mean free path is long compared to the Debye length, the Bohm condition will be appropriate. Thus the Bohm condition is a very general one for the onset of sheaths in plasmas.

Usually in a dc discharge, the current density is much greater than the ion current inferred from the Bohm condition, and the current going into the sheath from the plasma side is mostly electron current. At the cathode, if there were no secondary electron emission, the current would have to be entirely ion current. The question then is how the current transitions in the sheath, from ion current above the Bohm value at the cathode, to electron current in the main discharge. These are much more difficult aspects of the cathode fall, and explanations of them are qualitative.

Cobine¹⁴, and Lieberman and Lichtenberg¹³ both give qualitative explanations based on an essentially fluid description of the electrons in the cathode fall. Consistent with their fluid treatment, Refs. (13 and 14) discuss only ionization by the thermal electrons of the plasma. However, at least in the enhanced glow regime, an important cause of the ionization arises from a beam of electrons generated at the cathode and accelerated as they cross the sheath potential. On the other hand, the simulations of Refs. (7-12) consider ionization only from beam electrons, and not from thermal electrons. It finds qualitatively correct results for both hollow cathode discharges and also for discharges with planar electrodes. Clearly ionization by both thermal and by beam electrons may be playing a role. We give a somewhat different, but equivalent qualitative explanation here, one which leads into a similar analysis of the enhanced glow regime. However like Refs. (13 and 14), our theory is much more qualitative than quantitative.

There are two basic relations we make use of here. The ion current in the sheath is space charge limited, so if the ions are collisionless, the relation between voltage, sheath width and current density is given by the Langmuir Child's law for an ion diode:

$$J_{LC} (A/cm^2) = 10^{-8} (M_a/M)^{1/2} V^{3/2} (\text{volts})/s^2(\text{cm}) \quad (4)$$

where M_a is the mass of an argon atom, M is the ion mass and s is the sheath width. For a collisionless ion sheath, the electric field is proportional to $x^{1/3}$, where $x < s$ is the distance to the sheath boundary at the plasma. On the other hand, if the ions are collisional with constant mean free path, the diode relation is given by

$$J_\lambda = 2.3 (\lambda/s)^{1/2} J_{LC} \quad (5)$$

where λ is the mean free path of the ions. For this collisional ion diode, the electric field is proportional to $x^{2/3}$. Another idealization for the collisional ion diode is constant ion mobility. In this case, the diode relation becomes

$$J_v = (81/64)[(2eV/M)^{1/2} /sv] J_{LC} \quad (6)$$

where v is the (constant) collision frequency. For this diode, the electric field is proportional to $x^{1/2}$.

None of the diode laws replicate the experimental result that the electric field is proportional to x . The closest is the collisional ion diode law with constant mean free path. Furthermore, the measured cathode fall thickness is almost always quite large compared to the ion charge exchange mean free path, and the charge exchange cross section, as a function of energy, is much closer to being constant (constant mean free path), than it is to depending on energy as $E^{-1/2}$ (constant mobility). Thus the collisional ion diode with constant mean free path seems to be the best model, for the voltage drop in the cathode fall. However this model, like the others does not account for electron space charge, and more importantly, for ionization in the sheath. In the next section we will show how ionization in the sheath can give a better estimate for the case of ionization caused by an electron beam.

The ion diode relation cannot be the entire story because it involves three variables, V , J and s , whereas an Ohms law relates only V and J . Thus we need one additional relation. This comes from the requirement that ion current at the cathode has to transition to electron current on the plasma side of the sheath. In a steady state, one dimensional plasma, the total current is constant, but electron current may be exchanged for ion current. If we ignore deionization in the sheath, the conservation equations for electron and ion number density give

$$[d/dx]J_e = \alpha j_e + \sigma_i J_b N = - [d/dx]J_i \quad (7)$$

where α is the thermal ionization rate which depends on the appropriate integral over the electron distribution function of the ionization cross section. This rate is calculated from a solution of the Boltzmann equation, and/or are measured experimentally. In the limit of small electron population relative to neutral background, α/N depends only on E/N , the electric field divided by the neutral density, (or equivalently, the neutral pressure p).

Furthermore, we have assumed that the ionization is produced not only from thermal electrons, but also from beam electrons with current J_b . In order to precisely make this separation, it is necessary to define at what energy the thermal electrons end and the beam begins. One possibility, as in Refs.(7-12) is to define the beam as anything over the ionization energy. Another possibility, which we use, is to define the beam as a particle with an energy a several times the ionization energy. In this way, we can make simple contact with the data for ionization rates of thermal electrons¹³⁻¹⁵, so the enhanced ionization from the beam is over and above this. We consider the two limiting cases of ionization only by thermals, and only by the beam.

A. Ionization Only by Thermals

Reference (13) gives an approximate expression for the first Townsend coefficient

$$\alpha = A p \exp(-B p/E) \quad (8)$$

where α is in units of cm^{-1} , E is the electric field in V/cm , and p is the pressure in torr. The additional constants, A and B depend on the gas. For instance in air, Lieberman and Lichtenberg quote the numbers $A = 14.6 \text{ cm}^{-1}$ torr and $B = 365 \text{ V/cm torr}$.

It is assumed that when an ion strikes the cathode, it might liberate an electron. The quantity γ denotes the probability that an electron will be ejected from the cathode for each ion that strikes it. The quantity γ is small, usually around 0.05- 0.1. Thus an electron current γJ_i is emitted from the cathode. As is apparent from Eq.(7), this electron current will grow approximately exponentially, until at the plasma edge of the cathode fall, all the current is electron current. (It is not difficult to show that this exponentiation of electron current is entirely equivalent to assuming that each electron emitted from the cathode generates γ^1 electrons as it crosses the sheath. In this paper we will consider either current multiplication, or electron multiplication, whichever is more convenient.) If we assume that 3 e foldings are needed to produce the full electron current, and if we approximate E in Eq.(8) by its average value V/s , we find that

$$3 = 14.6 V Q \exp(-(365Q)) \quad (9)$$

where $Q = sp/V$. We note that the right hand side of Eq.(9) is V times a function of only Q . This function of Q is zero for both $Q=0$ and for $Q=\infty$. It has a maximum at $Q=1/365$. Correspondingly there is a minimum value of V for which the equation can be satisfied, and this minimum value is at $Q=1/365$. We, at this value of Q , find that $V=200$ Volts, $sp = 0.54$ torr cm, and using the Eq.(5) with $\lambda/s = 0.02$, the current density is $J/p^2 = 3 \times 10^{-5}$ Amps/(cm torr)². References. (13 and 14), for nitrogen and a variety of cathode materials, quote values of V between about 200 and 250 Volts, sp of about 0.3-0.4, and current density of $2-5 \times 10^{-4}$ Amps/(cm torr)² at the voltage minimum. The simple theory gives reasonably accurate estimates of V and sp ; the current density, which depends on

high powers of these parameters in the ion diode relations, is not calculated as accurately. Lieberman ¹³ actually does a somewhat more accurate calculation in which he assumes that E is linear in x . The results, nevertheless are not very different or more accurate than those given here. However it is worth noting, that if one uses experimentally measured values of V and s_p in the ion diode relation, the calculated value of J is more accurate.

The solution we have just estimated corresponds to the minimum voltage at which a cathode sheath can be sustained. It is characterized by a particular sheath thickness and current density at given pressure. As one tries to lower the voltage by reducing the current, one cannot. Instead the current decreases, at the specified current density, while the emitting area on the cathode surface correspondingly decreases. This is the normal glow. As the voltage increases from the minimum, Eq. (9) has two possible solutions. Clearly the only physical one is that for Q decreasing from 1/365. On this solution the voltage increases, s decreases, and the emitted current increases. This is the abnormal glow, where the emitting surface covers the whole cathode and the current density increases with voltage. Its current voltage relation is determined by the ionization relation and the appropriate ion diode relation.

B. Ionization Only by the Beam

As each ion strikes the cathode, it liberates γ electrons which are accelerated by the sheath field and ultimately form a beam. Thus the equation for the electron current in the sheath becomes

$$dJ_e/dx = \sigma_i J_b N = \gamma \sigma_i J_o N \quad (10)$$

where J_o is the plasma current, assumed much larger than the beam current. The primary ionization cross section, σ_i for say Nitrogen, reaches a maximum of about $3 \times 10^{-16} \text{ cm}^2$ at an energy of about 100 ev, and between about 50 and 500 ev, it is above $2 \times 10^{-16} \text{ cm}^2$. (Since the beam electron energy is fairly low, a few hundred volts, we neglect the ionization generated by the progeny of the beam electrons.) Thus over the energy range of interest in the cathode sheath, we can approximate σ_i as constant, say about $2.5 \times 10^{-16} \text{ cm}^2$. If we assume that $\gamma = 0.1$, Eq. (10) indicates that the ion current transitions completely into electron current in a distance of about

$$Ns = 1/\gamma \sigma_i \approx 1 \text{ torr cm} \quad (11)$$

which is roughly the same as, but slightly larger than what was calculated in the last subsection. A minimum voltage does not arise directly from Eq.(10), but rather from the energy needed by the beam to produce this current. Each ion takes about 15 ev to produce, and ionization accounts for approximately half of the energy loss of the beam electrons. In crossing the sheath, each beam electron has to produce $1/\gamma$ beam electrons. Thus the minimum voltage across the sheath is about $30/\gamma$, or about 300 ev for our assumed value of $\gamma=0.1$. This is also qualitatively correct, although not as precisely

specified as the voltage minimum for the case of ionization from thermal electrons only. Also, as we will see in the next section, accounting for ionization in the sheath by the beam electrons increases the diode current. Thus both beam ionization and thermal ionization give qualitatively correct values for the minimum voltage and the sheath width. Addition of beam ionization to thermal ionization would increase the total ionization, and thereby reduce the predicted value of s_p calculated in the previous subsection.

III The Cathode Region of the Enhanced Glow

There are two remarkable aspects of the enhanced glow discharge. The first is the dependence on cathode shape. Two cathode shapes are shown in Fig. (1). They are called the shallow and deep cathodes, and their widths are $D = 1.6$ and 1.2 cm respectively, and their heights are $h = 1.2$ and 2.4 cm respectively. The former runs at high voltage (above 2 kV) and higher impedance. The second runs at low voltage and lower impedance. The former produces an electron beam which can be detected at the anode, and produces a plasma which reflects X band microwaves; the latter does neither. (However the latter also cannot run at the voltages at which the former generates the beam.) The second aspect is that the discharge is steady state (ie no current runaway). The discharge of Ref. (5) is run with an 80Ω series resistor. However the impedance of the enhanced glow shallow cathode turns out to be considerably larger than this. The deep cathode, on the other hand, has low enough impedance that the current is determined by the series resistor as well as the plasma.

We assume that there is a thin (thickness much less than cathode dimension) sheath hugging the walls of the cathode and that there is a uniform plasma outside this sheath, but within the cathode hollow. Electrons emitted from the cathode accelerate through the sheath and form a beam which bounces around through this uniform plasma in the cathode hollow. In bouncing around there, they generate enough ionization to maintain the plasma. Since the density of beam electrons in the hollow is surely greater than the beam electron density in the main agile mirror plasma, we expect the electron density to be larger in the cathode hollow than in the main agile mirror plasma. Since recombination is important in the main plasma, it will be even more important in the cathode hollow. However, as the beam electrons bounce through the sheath, they also cause ionization there, and this ionization is what is assumed to give the current multiplication which maintains the sheath. The remainder of this section considers important aspects of the theory, the uniform plasma assumption, the ion diode Child Langmuir law including beam ionization, and the scaling laws for the enhanced glow.

A. The Uniform Plasma Assumption

One assumption we make is that the plasma in the cathode hollow is uniform, and also that the cathode sheath is narrow and hugs the cathode shape a small distance from the cathode. If the plasma is uniform, the Bohm condition dictates that the ion current is uniform around the cathode, and thus the sheath thickness is uniform as well. For the voltages, currents and sizes of the cathode of the NRL agile mirror experiment, the distance is so small (about a millimeter) that the collisionless ion diode law is the appropriate one to apply. As pointed out in Ref.(5) the distinction between the abnormal glow and the enhanced glow (for the shallow cathode) or the hollow cathode mode (for the deep cathode) is that the sheath either hugs the cathode or is relatively unaffected by the shape of it. As one varies parameters in going from one regime to another, the transition is, in some cases, very sudden. There have been simulations of this sudden

transition as well.¹⁶ We will not be concerned with the transition here, but will consider plasmas only having the sheath hug the cathode.

Let us now consider the effect of the magnetic field on the assumption of uniformity. The experiment has a magnetic field of typically 100-200 G running from cathode to anode. Electrons emitted off the side wall will be deflected by this field. As long as the Larmor radii of electrons overlap, ie $r_L > D/2$, where D is the cathode width, the plasma produced by this beam should be reasonably uniform. For larger fields, the electrons may return to the original electrode from which it was emitted because of magnetic deflection; for smaller fields it may be reflected from the opposite sheath. However this will not affect the assumptions or the theory developed here. As long as the field is below some critical value, the voltage current characteristics of the cathode generally do not depend on the magnetic field.¹⁷ The charge collector data⁵ which detects the electron beam shows a beam with a peak down the center of the sheet plasma. However while the cathode's voltage current relations is predicted to be independent of magnetic field, this is not so of the plasma itself. To produce an effective agile mirror plasma, the beam electrons must be confined in the main plasma. These beam electrons suffer small angle collisions with the neutrals. If there were no magnetic field, we will see shortly that they would spread out considerably laterally; the beam thickness would increase as roughly the distance from the cathode to the three halves power. This would considerably reduce the electron density in the agile mirror plasma and render it ineffective as a microwave reflector. The minimum magnetic field then is determined by the requirement that the larmor radius of the electron, with a velocity perpendicular to the magnetic field equal to that which it would obtain from the cumulative multiple small angle scattering upon reaching the anode, is less than the mirror plasma width. Below that field the mirror plasma spreads out and is not effective in reflecting X band microwaves.

On the other hand, if $r_L < D/2$, the beam electrons do not reach the middle of the cathode hollow, and there is a region, whose width increases with magnetic field, where no plasma is produced. Correspondingly, as the field increases, there is a region of the back wall of the cathode from which no electron are emitted. This is also observed in the experiment.¹⁷ As the field increases, the plasma ultimately approaches two plasmas, one emitted from each side wall. This is visually apparent.

We now consider what makes the sheath uniformly hug the cathode, as illustrated in Fig. 2a. If we think of the potential in the hollow as a solution of the vacuum Laplace's Equation, it is clear that the inner parts of the diode are shielded in a geometry dependent way. Figures (2 b and c) show schematically what an equipotential for the vacuum solution to Laplace's equation might look like for the two cathodes used in Ref.(5). In the deep cathode, the back wall is shielded much more than it is in the shallow cathode; that is the s in any of the diode relations are considerably larger, and is also considerably large than for the side wall. The current density coming from the back wall will be correspondingly less (recall that the current density scales as something between s^{-2} and s^{-3}).

Unfortunately, the vacuum solution of Laplace's equation is not correct in the presence of a plasma. Recall that the condition for the breakdown of quasineutrality is the Bohm condition and this is a very fundamental requirement. As long as the density, temperature, and current density are reasonably uniform along the edge of the cathode hollow, the conditions for formation of the sheath are also uniform throughout the hollow. In this case, we would expect s to be nearly uniform all around. That is, the plasma tends to pull the equipotential into the cathode hollow until it is nearly up against the cathode.

Nevertheless, it does seem as though there may be some effect of both variation in plasma parameters as well geometric shielding of both the potential surface and plasma production, even though it would be much less than it would be for the vacuum potentials. The only reliable way to calculate this would be with a numerical simulation like those in Refs.(7-12). In fact, these calculations do in many cases find a difference in shielding, although in these simulations, neither the Bohm criterion nor ion diode relation is correctly represented (more on this in the next section). Shown in Fig.(2d) is a sketch of an equipotential surface from the Ref.(7) for pseudospark configuration. There is clearly a geometric shielding of the upper and lower surfaces in the rear box. The sheath thickness there are about a factor of 3 larger than they are at the back surface. This would correspond to at least an order of magnitude less current density. If this same shielding were to hold true for the deep cathodes, it, as well as the lower voltage operation, might explain the absence of the electron beam in the agile mirror deep cathode experiments. A more quantitative understanding of this phenomena will have to rely on numerical simulations of Poisson's equation in the cathode hollow region.

B. The Langmuir-Child Ion Diode Including Beam Ionization

Here we develop a more accurate model of the ion diode which describes the sheath. The sheath is a few millimeters thick, and is comparable to or thinner than the ion charge exchange mean free path. Hence it is reasonable to assume the ions are collisionless in transiting the sheath. To review, we write once more the Langmuir-Childs expression for the current density of an ion diode. It is

$$J = V^{3/2} (2e/M)^{1/2} / 9\pi s^2 \quad (12)$$

where now all units are cgs.

In the enhanced glow cathode, while the ions are nearly collisionless, there is also a density of beam electrons reflexing back and forth in the sheath. This causes ionization, and as we discussed in the last section, this ionization causes the ion current at the cathode to transition to electron current at the entrance to the sheath. As the electron-ion pairs are generated, the electrons are accelerated through the sheath and into the main plasma. Since they are so much lighter than the ions, and their current is comparable to the ion current, their space charge is negligible compared to the ion space charge.

Let us say that the beam gives rise to a uniform ionization rate, so that the equation for the ion current density is

$$dJ_i/dx = K \quad (13)$$

Since the electron current production is negative this, the total current in the sheath is uniform, while the electron and ion currents have spatial dependence. Let us say that the ion current entering the sheath is zero, so that as a function of position in the sheath, the ion current density is given by $J_i = Kx$. Hence at the cathode, at $x=s$, the total current density is equal to that of the ions, or

$$J = Ks \quad (14)$$

At a position in the sheath between x_0 and x_0+dx_0 , Kdx_0 electron-ion pairs are produced. The ions produced here accelerate toward the cathode. However ions produced at different x_0 give rise to different space charge at position x , because they have different velocity depending on where they were produced. The analogous equation for the ion Langmuir-Child diode becomes

$$d^2\phi/dx^2 = 4\pi(M/2e)^{1/2} \int_0^x dx_0 K/[\phi(x) - \phi(x_0)]^{1/2} \quad (15)$$

Equation (15) is a nonlinear integro-differential equation for the electrostatic potential. Amazingly, it has a simple analytic solution. By inspection, one can show that if the electric field is linear in x , or $\phi(x) = V(x/s)^2$, both sides are independent of x , and Eq.(15) reduces to an equation relating the various parameters, J , V and s . Making use of Eq.(14), one can show that

$$J = (2e/M)^{1/2} V^{3/2} / \pi^2 s^2 \quad (16)$$

Thus Eq.(15) does have a simple analytic solution. It predicts that the electric field in the diode varies linearly with x , which is in better agreement with most experiments than the diode laws from the last section. Also it predicts the Langmuir-Childs scaling, but with a current density approximately a factor of three larger than the conventional collisionless diode, Eq.(12).

It is also interesting to point out that analogous relations can be derived for the collisional diodes with constant mobility or constant mean free path. In each case, the scaling of current density with voltage and s is as before (that is Eqs.(5 and 6)); however, as in the collisionless case the current density is about a factor of 3 larger. In the case of constant mobility, E is also proportional to x , while for the case of constant mean free path, it is proportional to $x^{4/3}$.

C. Approximate Enhanced Glow Relations

We assume that the electrons emitted from the back wall of the cathode free stream into the main agile mirror discharge and create the plasma there. The electrons emitted from the side walls bounce around in the cathode hollow until they are scattered out. If there were no collisions, self fields or geometric imperfections, the electrons would bounce back and forth indefinitely. However, because of the collisions, the electrons scatter out of the cathode hollow. Let us define the parameter B as the number of bounces an electron makes back and forth before it is scattered out of the cathode hollow region. Since the scattering is a succession of many small angle deflections for energetic electrons, the deflection angle of the electron is given by

$$d\theta^2/dz = 2N\sigma_M(V) \quad (17)$$

where N is the neutral density and $\sigma_M(V)$ is the momentum exchange collision frequency as a function of beam voltage V . Since $\theta = v_1/v_z$, we can integrate Eq.(17) to get the perpendicular distance a typical electron travels as it undergoes a succession of small angle scattering,

$$r_\perp \approx z^{3/2} (8N\sigma_M/9)^{1/2} \quad (18)$$

Let us consider that the beam of electrons spreads out and has a radius in the transverse plane given by Eq.(18). If an electron starts at the top of the cathode, it is out when it goes a downward distance of h . (This should be a reasonably accurate estimate; if it starts half way up, it must go a distance of $h/2$ if it starts going down, and a distance of $3h/2$ if it starts going up. Let us estimate B by assuming that if the radius of the half circle of electrons, $\pi r_\perp^2/2$ is twice the area of the circle included in the hollow cathode, $2h r_\perp$. This gives the result

$$B \approx 2h^{2/3} / D(N\sigma_M)^{1/3} \quad (19)$$

Now, for the electron momentum exchange cross section, we will use the approximation

$$\sigma_M(V) \text{ cm}^2 = 8 \times 10^{-18} [E(\text{kev})]^{-1.7} \quad (20)$$

For the cathode dimensions, voltages (about 3 kev), and gas densities ($N \approx 3 \times 10^{15} \text{ cm}^{-3}$) characteristic of the agile mirror plasma, is B is typically about 9. This diffusion in perpendicular dimension should help to create the reasonably uniform plasma in the cathode hollow which we have been assuming. Using this expression for momentum exchange cross section, we find that

$$B \approx 10 h^{2/3} V^{1.7/3} / D \eta \quad (21)$$

where as usual h and D are in cm, V is in kilovolts, and η is the background number density in units of 10^{15} cm^{-3} .

Now let us consider the ionization which the beam electron creates as it crosses the cathode. If the primary electron emitted from the cathode travels through the sheath on each side B times, the number of secondary ions and electrons it generates in each sheath is

$$\Pi = B \int N \sigma_i(V) dz \quad (22)$$

where the integral is through the sheath. To approximate this, we need the ionization cross section as a function of energy, and the voltage (ie beam energy) in the sheath as a function of position. For the ionization cross section, we take a simple expression which is reasonably accurate for N_2

$$\begin{aligned} \sigma_i(V) &= 10^{-16}/V & V > 0.3 \\ \sigma_i(V) &= 3 \times 10^{-16} & V < 0.3 \end{aligned} \quad (23)$$

where the ionization cross section is in cm^2 and the voltage (energy) is in kilovolts. We assume, consistent with the derivation in Sec III.B that the sheath has an electric field which increases linearly from the edge of the sheath to the cathode over a distance s , so the Voltage as a function of distance is

$$V(z) = V [1 - (z/s)^2] \quad (24)$$

where now V is the cathode voltage and s is in centimeters. Doing the integral, we find that Π has a dependence on V going as V^{-1} times a logarithm of V . Neglecting this latter dependence and evaluating V in the logarithm at 3 kV, we find

$$\Pi \approx 0.23 B \eta s/V = 2.3 h^{2/3} \eta^{2/3} s/D V^{0.43} \quad (25)$$

These secondaries are themselves accelerated through the sheath potential and produce tertiaries, etc. The number of tertiaries is a complicated convolution involving which energy the particular secondary was produced at. Making the simplest assumption, that all electrons act the same, one can get the total number produced by summing up the geometric series.

$$1/\gamma = 1/(1-\Pi), \text{ or } \Pi = (1-\gamma) \quad (26)$$

This then allows us to solve for the sheath thickness,

$$s = 0.4 D V^{0.43} / h^{2/3} \eta^{2/3} \quad (27)$$

For our canonical parameters, $D=1.6$, $h=1.2$, $V=\eta=3$, we find that s is about 0.4 cm. Assuming that the current is determined by Eq.(16), we find an expression for the ion current as a function of voltage, pressure and geometric parameters. The electron current

density, J_e in the main plasma is actually a more accessible measurement, and it is larger than the ion current density by a factor of $(2h+D)/D$. Inserting this and s from Eq.(27) into Eq.(16), we find

$$J_e(\text{A/cm}^2) = 4.5 \times 10^{-3} V^{0.64} (h\eta)^{4/3} (2h+D)/D^3 \quad (28)$$

For our canonical parameters, we find $J_e = 5 \times 10^{-2} \text{ A/cm}^2$, which does not agree too badly with the measured result.

Let us now consider other dependences. The current density depends on the geometric factors of the diode as $h^{4/3} (2h+D)/D^3$. Comparing these factors for the deep and shallow diode, we see that the current density for the deep diode is about a factor of 8 larger than for the shallow one. Thus the deep cathode is predicted to be a much lower impedance cathode. The measured impedance of the shallow cathode at canonical parameters is about 250Ω , so the 80Ω load resistor has only a small effect on the total current. The load resistor will be a much more dominant part of the total resistance for the deep cathode however, and because of its presence, high voltage operation was not possible for the deep cathode in the NRL experiment.

Equation (28) predicts the current scales with $V^{0.64}$. This is qualitatively correct in that the current is an increasing function of current; however the measured increase of current with voltage in Ref. (5) is faster, it is at least linear in voltage. The dependence of current density on pressure scales as the pressure to the four thirds power. This scaling seems to be in reasonable agreement with the results quoted in Ref.(5) for the shallow cathode. Shown in Fig.(3) are plots of current density versus voltage at $\eta=3$, and of current density versus pressure at 3 keV. To summarize, we have developed here a qualitative theory for the cathode behavior in the enhanced glow regime of the NRL agile mirror plasma. The theory gives reasonable qualitative agreement with the experiment and predicts scaling rules of current on voltage, pressure and geometry which are also in qualitative agreement with the experiment. It seems likely that better quantitative agreement would have to come from a numerical simulation of the plasma of the type done in Refs.(7-12).

IV The Possibility of Numerical Simulations of the Enhanced Glow Cathode

The closest existing numerical simulation scheme for the enhanced glow cathode appears to be the series of codes developed by Boeuf and Pitchford.⁷⁻¹² Their codes have been developed to model hollow cathode plasmas such as pseudo sparks. These codes use a fluid model for electrons and ions where the inertia of each species is neglected. Thus the velocity of the electrons and ions are given by

$$v_e = -\mu_e E - n_e^{-1} D_e \nabla n_e \quad (29)$$

$$v_i = \mu_i E - n_i^{-1} D_i \nabla n_i \quad (30)$$

where the D's and μ 's are positive and are related by $D = T\mu/e$ for each species. The D's, μ 's and T's are specified in terms of the local value of E/p, as given for instance in Dutton¹⁵. From the velocities, the electron and ion densities are obtained from the continuity equations. The densities are coupled to the electrostatic potential through Poisson's equation.

Ionization is accounted for in a separate Monte Carlo calculation. For every ion striking the cathode, γ electrons are liberated, and their orbits are followed, their ionization and excitations are calculated, and where necessary, their progeny are also followed. The energy boundary between Monte Carlo electrons and thermal electrons was taken to be the ionization energy, so only the Monte-Carlo electrons can cause ionization. While this is certainly correct, it may also be that one would want to consider an alternate scheme where the boundary is somewhat higher and ionization from thermals were also considered. The thermal ionization rate would be a specified function of E/p as in the case of D, μ , and T. This could be simpler in some cases, especially if there were other sources of electrons above the ionization energy, for instance electron-electron collisions or Ohmic heating. Alternatively, if the ionization energy were retained as the boundary between the Monte Carlo and thermal electrons, then some source for Monte Carlo electrons from the thermals should really be included. This could be complicated, it might well be easier to set the boundary somewhat above the ionization energy. However either way, the basic simulation appears to be a mostly viable one for the enhanced glow plasma.

Because elastic scattering of electron at the high energies appropriate to the agile mirror plasma, is multiple small angle scattering rather than hard sphere binary collisions, it is best to follow the electron scattering with a Langevin equation rather than a binary collision model.^{18,19} Also, since the ionization in the sheath is so important, the motion of the Monte-Carlo electrons in the sheath, as well as the sheath itself must be accurately resolved.

There do appear to be some modifications to these codes which could aid in giving a more accurate simulation. These are mostly the inclusion of ion inertia, inclusion of the magnetic field, and inclusion of recombination. This latter effect is particularly important in the agile mirror, where even in the main mirror plasma, recombination is important. In the cathode region, with a higher beam and plasma density, it will be more important still. Even in some of the published results,⁷⁻¹² in some cases the electron densities look sufficiently high that recombination should be included. Inclusion of recombination appears to be a simple modification to the simulations of Refs.(7-12). The magnetic field is also important in the agile mirror plasma. Depending on the pressure, the magnetic field might or might not have an effect on the fluid motion. However it would certainly have an effect on the beam electrons. It probably would not be very difficult to include the effect of the magnetic field on both the fluids and also on the Monte-Carlo electrons in the simulations of Refs. (7-12).

We now discuss the neglect of ion inertia. This is very important in at least two respects. First of all, we have seen that the current voltage relation depends critically on the ion Langmuir Child's diode law. The simulations of Refs.(7-12) are only capable of modeling a mobility limited diode, whereas in fact the sheath thickness, (a few millimeters), is comparable to the ion charge exchange mean free path (charge exchange cross sections for N_2^+ in N_2 at the relevant energies are about $1.5-3 \times 10^{-15} \text{ cm}^2$). Probably the best model for the ion diode in the sheath in the cathode hollow is the collisionless one.

The second aspect involves the Bohm condition and the breakdown of quasi-neutrality. As we have seen, the Bohm condition depends very directly on the inclusion of ion inertia. Collision terms modify this, but these are only important if the collision mean free path is comparable to a Debye length, a condition which rarely occurs in practice. Let us consider the analog of the Bohm condition where the electron and ion motion is governed by Eqs. (29 and 30).

Instead of examining perturbations to an equilibrium, we look at the one dimensional equilibrium itself. Assuming cold ions (no direct ion diffusion), the equations for the equilibrium are

$$\frac{d}{dx}[n_e(-\mu_e E - n_e^{-1} Ddn_e/dx)] = S \quad (31)$$

$$\frac{d}{dx} n_i \mu_i E = S \quad (32)$$

$$\frac{dE}{dx} = 4\pi e(n_i - n_e) \quad (33)$$

where S is the ionization source.

We now examine whether there can be a quasi-neutral (ie $n_i = n_e \equiv n$) solution to this equilibrium. In one dimension, the current $J/e = -n_e v_e + n_i v_i$ must be constant. Thus in terms of the current,

$$E = -[-J/e + Ddn/dx]/n(\mu_e + \mu_i) \quad (34)$$

or

$$-d/dx D[\mu_i/(\mu_i + \mu_e)] dn/dx = S \quad (35)$$

There are two cases one might consider, $S = \text{constant}$, (ionization by an external beam), or $S = Kn$ (ionization by the thermal electrons). Let us also say that the boundary condition is that the density vanishes at $x = \pm a$. For the former, the density is parabolic with the central density determined by S . For the latter, the density is proportional the cosine of a constant times x . The constant is determined by the ionization rate and the central density is not specified. From the calculated electric field, one can calculate the actual charge separation from Eq.(28). As long as

$$(4\pi e)^{-1} dE/dx / n \ll 1 \quad (36)$$

the plasma is quasi-neutral. If the electron mobility and diffusion are related by the standard relation, $D = T_e \mu_e / e$, one can show by inspecting the various terms when Eq.(34) is substituted into Eq.(36), that the dominant effect relating to quasi-neutrality is the standard condition

$$[T_e/4\pi e^2 n^2] d^2 n/dx^2 \equiv \lambda_D^{-2} n^{-1} d^2 n/dx^2 \ll 1 \quad (37)$$

where λ_D is the Debye length. Thus the plasma is quasi-neutral until one gets to very small n near the wall. At this point a sheath with charge separation forms. Hence the mobility limited flow model so far gives a reasonable result; namely that the bulk of the plasma is quasi-neutral, but that there is a region near the wall, of size λ_D , where charge separation is important.

However the expression for the flow speed at which quasi-neutrality is violated is not correct. Neglecting electric current, and we can calculate that the ion flow speed is given approximately by

$$v_i = -n^{-1} D[\mu_i/(\mu_i + \mu_e)] dn/dx \quad (38)$$

If we assume $\mu_i \ll \mu_e$, $\mu_e = e/mv_e$ and $v_i = (m/M)^{1/2} v_e$, we find that

$$v_i = (T_e/M)^{1/2} \times (T_e/M)^{1/2} (v_i L_n)^{-1} \quad (39)$$

where L_n is the density gradient scale length. If we assume that quasi-neutrality is violated when $L_n = \lambda_D$, then the condition on the ion velocity is that quasi-neutrality breaks down when the ion fluid has accelerated to a velocity

$$v_i = (T_e/M)^{1/2} \times (\omega_{pi}/v_i) \quad (40)$$

where ω_{pi} is the ion plasma frequency. Since almost always, $\omega_{pi} \gg v_i$, the flow speed predicted for the breakdown of quasi-neutrality is considerably larger than the actual value, $v_i = (T_e/M)^{1/2}$. Thus, while the mobility limited flow model does predict quasi-neutrality as long as the gradient scale length is long compared to the Debye length, and charge separation for shorter scale lengths, that is quasi-neutrality in the bulk and charge separation near the wall; it gives an incorrect estimate of the ion flow into the non-neutral sheath.

The question is how important are these errors to the Bohm condition and ion diode law. It may be that the ion flow pattern and electric currents are entirely determined by bulk processes. Then the current density and flow into the sheath (and thereby the cathode) is determined entirely by the bulk processes, and the appropriate sheath just sets itself up to respond. While the details of the sheath may be incorrect, everything else would be properly described. On the other hand, accurate descriptions of the flow and current near the wall could be very important, necessitating inclusion of ion inertia. Either way, it would be better if ion inertia could be included. Unfortunately this is difficult to do. It may be that at least initially, the simulations would have to provide whatever insight they could with mobility limited ions.

To summarize, it seems as though the simulations of the type done in Refs.(7-12) could shed considerable additional insight on the enhanced glow mode. However, to be as effective a model as possible, it seems as though some modifications would have to be made in these codes, especially the inclusion of recombination, magnetic field, and ion inertia.

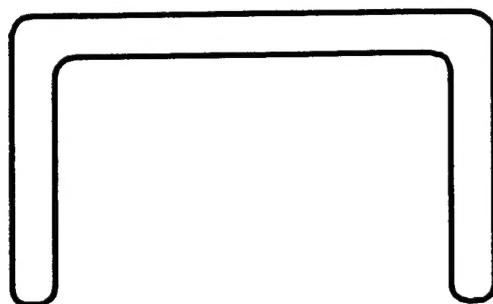
Acknowledgment: The authors wish to thank Leanne Pitchford, Robert Meger and Joe Mathew for a number of useful discussions. This work was supported by ONR.

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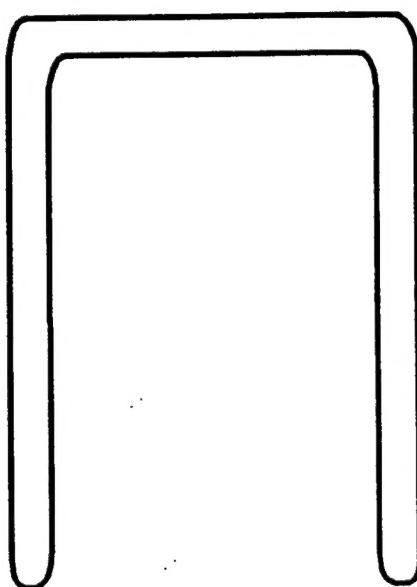
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a)



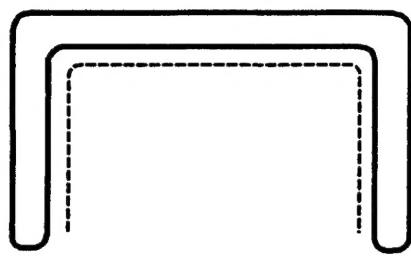
Shallow Cathode

b)

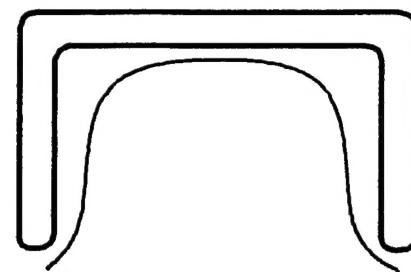


Deep Cathode

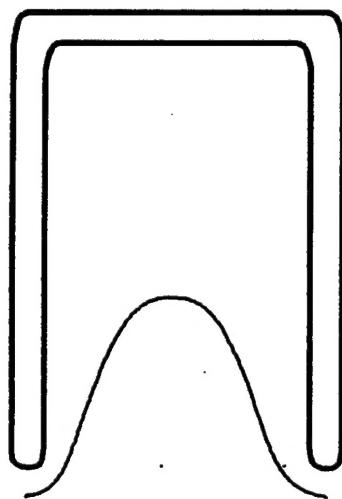
Figure 1 Cathode Shapes in the Experiment of Ref 5



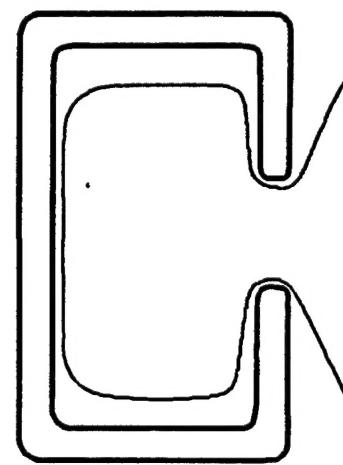
a) Cathode and Sheath



b) Vacuum Shielding of Shallow Cathode



c) Vacuum Shielding of Deep Cathode



d) Schematic of Geometric Shielding from Ref 7

Figure 2 Possible Cathode and Sheath Configuration

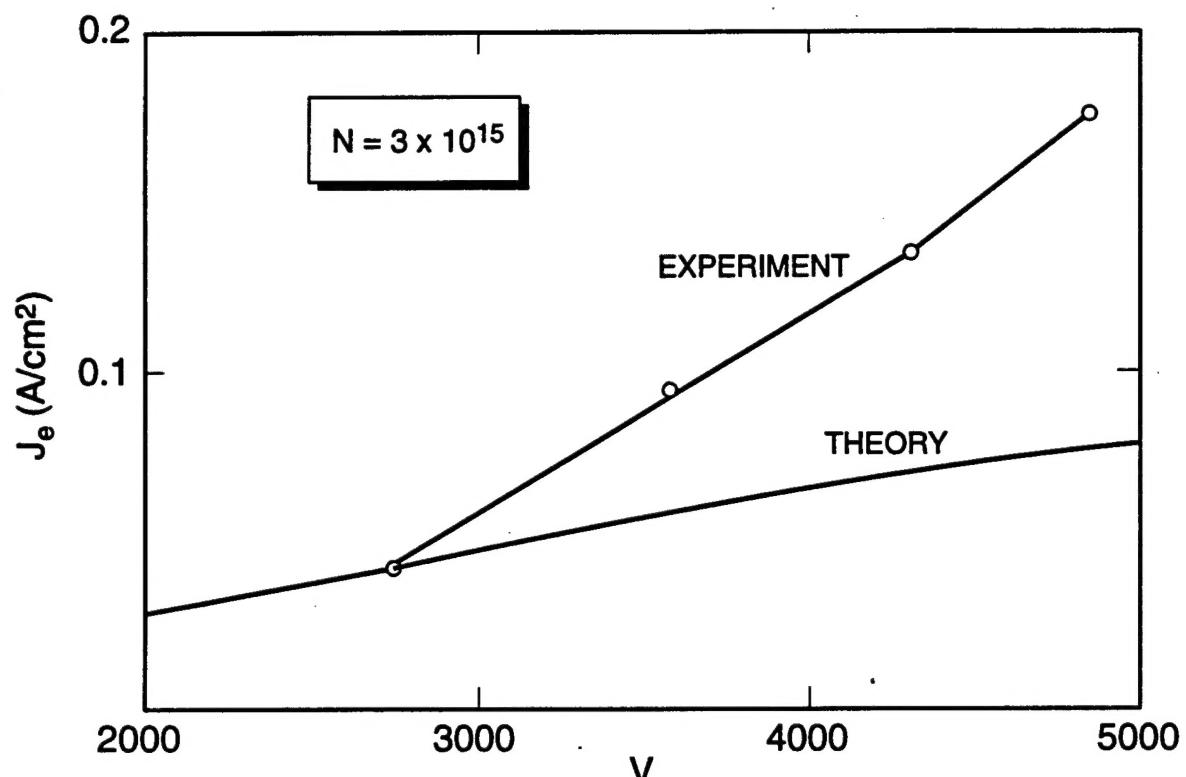


Fig. 3a - Current density vs voltage, experiment and theory

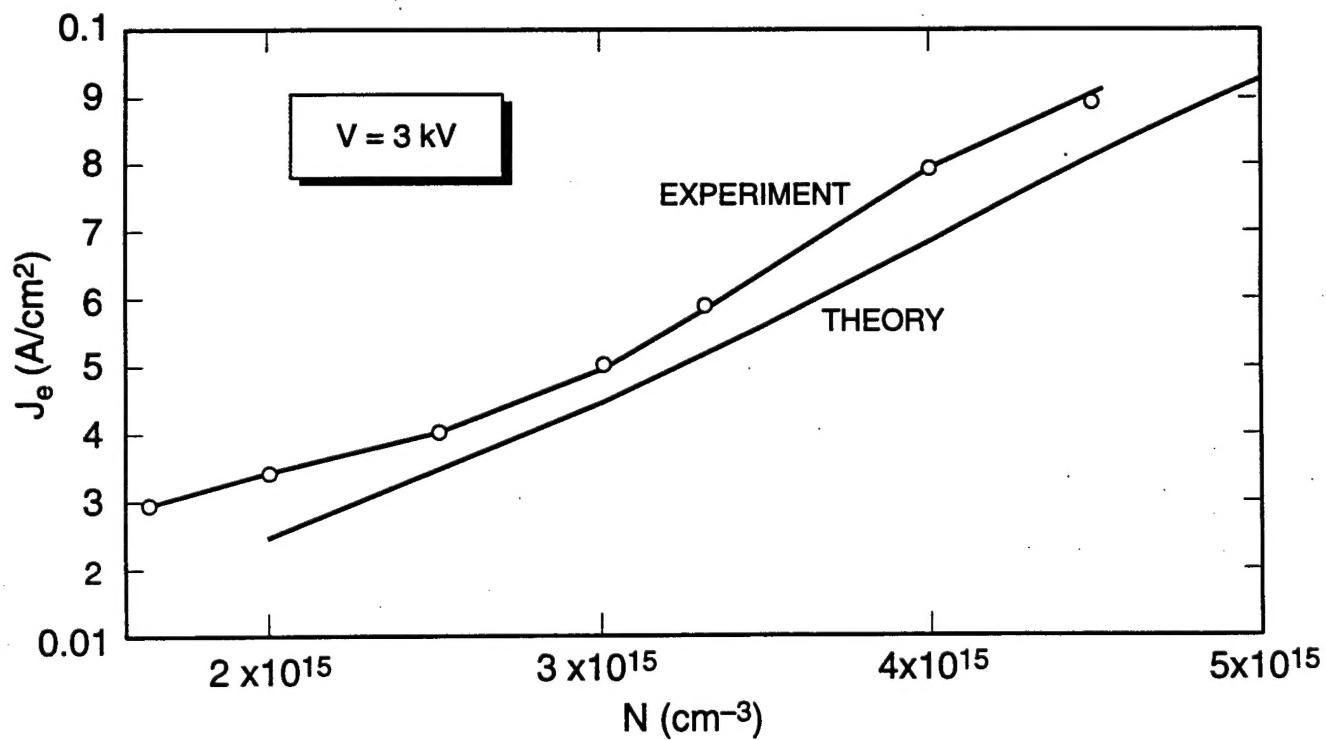


Fig. 3b - Current density vs gas density, experiment and theory